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Learners' Interpretation of Geometric Concepts in the Intermediate Phase: The Case of Limpopo Province

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ABSTRACT

This paper reports on an exploratory study in which questionnaires assessing the understanding of grade 4 mathematics concepts learnt as prescribed by Continuous Assessment Policy Statement (CAPS) were administered to 219 grade 5 learners from 20 randomly selected primary schools in the Limpopo province of South Africa at the beginning of the year 2014. This paper reports only on the understanding displayed in responses to five out of twelve questions that sought interpretation of geometric concepts in the questionnaires. This was a both quantitative and qualitative study in which structured and semi-structured interviews were conducted with six learners on the basis of their responses to gain more insight into how they constructed their geometric meanings. Results indicated that some learners did not understand the questions but would respond to questions based on a word or diagram that they associated with a particular concept. Nonetheless, 99% learners displayed understanding and familiarity with the concept of similarity. It can be recommended in this study that the interpretation in some questions be explained using African languages that were used in the learners' previous grades. Also the use models like photos, diagrams, graphs, symbols, icons, and other visual representations can help promote the conceptual understanding of geometric concepts to learners in the intermediate phase. The study therefore concludes that for learners to interpret geometric concepts in the intermediate phase, their geometric vocabulary should be developed such that they are able to make connections between new ideas and their prior learning.

Keywords: Geometry, Area, Squares, Symmetry, Triangles.

Introduction

Geometry is a component of mathematics which forms the bedrock of engineering and technological development and if taught with inadequate resources in its abstract nature, renders it difficult for learners to understand. Learners experience geometry throughout the foundation phase by identifying, comparing, sorting, and classifying two-dimensional shapes. They then learn and understand the defining properties of various two-dimensional shapes through activities that involve both examples and non-examples of those shapes in the intermediate phase. Learners in grade 4 usually learn geometric concepts through a variety of instructions, including manipulating concrete materials as a measure of discovery of properties of shapes and figures. For example rectangular geometric objects often used as examples in classrooms include books, walls, cupboard faces and doors. Meanwhile it takes a gradual process for learners to develop an understanding of experiential and visual learning to abstraction of knowledge about geometrical figures. These are components of spatial sense which learners must acquire, to have a deeper appreciation of the world that surrounds them. They can develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions. Geometry can be thought of as the science of shapes and space, while spatial sense is "an intuitive feeling for one's surroundings and the objects in them" (National Council of Teachers of Mathematics, 1989, p. 49). The geometry and spatial sense strand in Grades 4 requires the learners to learn (i) properties of two-dimensional shapes and three-dimensional figures, (ii)geometric relationships and (iii) location and movement (DOE, 2011).

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According to Piaget (1966), the origin of spatial representation coincides with the origin of drawing, language and representational thinking in general. He suggests that drawing should be conceived as the representation or the motor of spatial conception, since geometry is founded on the practice of drawing. Piaget (1972) advocates that Euclid's elements and the topological properties of shapes have their origin in cognitive schemes that every man and woman builds up in reflexive interaction with objects. Cognitive schemes in this context refer to structures of perception and reorganization of spatial conception as a result of children's interaction with their immediate world. These are representational images which children construct initially in their minds, refine, revise and transform into connected concepts as they are exposed to the physical world. On the expression of those drawings interpreting the world, geometry is founded.

Geometry is an important sub domain of mathematics which is frequently used in real life. Wang and Woo (2007) suggests a genetic approach to the teaching of Geometry where the teaching (i) follows a way of knowledge development, (ii) is connected to a way of cognition in mathematics, and (iii) follows relevance of logic. Genetic approach refers to a way in which the development of the contents of the mathematical theory can be explained (Safuanov, 2007). The author associates this approach with teaching based on previously acquired knowledge, experience and level of thinking of learners. Meanwhile a structured set of mental constructs which might describe how a concept can develop in the mind of an individual is known as the genetic decomposition of that particular concept, Dubinsky (1991). He further suggests that a proposed genetic decomposition should guide the instructional design for a particular lesson to ensure the formation of relevant mental constructs. Nonetheless, his notion of genetic decomposition is based on (Actions, Process, Object, Schema) APOS (Dubinsky, 1991).This is a theory that presupposes a transformation perceived externally in the form of an action before being processed and interiorized in an individual's mind. It would then be encapsulated as an object and the totality of the process would result in an individual forming schema of that particular concept. Contrary, Tall (2004) objected to this citing that in geometry learners first visualize the object as a totality before they can act on exploring its properties. Thus in geometry the learner will see a drawing as an object first before processing it.

Effandi &Rahim, (2012) assert that geometry is an important topic in mathematics and in any school curriculum. It is among one of the basic skills to be mastered and enables us to describe, analyze, and understand our physical world, (Gavin, Belkin, Spinelli, and St. Marie, 2001, p. 1). Beskin (1947) notes that, geometry must be shown to the learners not in a complete, crystallized rigid structure but in the process of development. In this way learners can be active creators of geometry rather than dealing with complete structures whose properties are not known to them. At the intermediate phase, learners can be exposed to constructions of geometric figures using concrete materials like match sticks or straws. This concurs with the fact that mathematical ideas begin with human activity and then proceed to be abstract concepts (Dubinsky and McDonald, 2001). It is therefore important for us to understand how the construction of concepts in the mind, lead to abstraction of mathematical knowledge (Jojo, Brijlall and Maharaj, 2013). The interpretation of the relevant knowledge construction processes is essential since it points to the contributions we get from the genetic approach. These include (1) understanding the importance of human thought, and (2) pointing to effective pedagogy for a particular concept (Jojo et al., 2013).

The genetic approach to the teaching of geometry subjects learners to a knowledge development process where they develop a habit of clear thinking and precise expression whilst learning. Geometry is an aspect of mathematics which deals with the study of different shapes which may be plane or solid. Government related, curriculum rebated, and examination body related variables, together with under qualified teachers, poor primary school background and textbook related issues have been implicated as responsible for the dismal performance of learners in mathematics. Telima (2011) associated the difficulty in understanding geometry with failure of the mathematics to relate to the children's environment where the children cannot see its importance and immediate application in their day to day living. This paper explores learners' interpretation of the geometric concepts in the intermediate phase.

Literature Review

For many learners mathematics is predominantly passive and rural especially when they have to sit and learn about triangles, decimal multiplication or algebraic manipulation (Pimm, 1987). On a contrasting note, (National Council of Teacher of Mathematics (NCTM), 2010) proposes that engaging learners in examining, measuring, comparing, and contrasting a wide variety of shapes develops essential learning skills in geometry. NCTM further emphasizes four reasons on why geometry should be studied. They mentioned that geometry: (i) forms the most important part of our synthetic world like cars, machines, art architecture and everything that humans can create, (ii) explorations can develop spatial reasoning which forms the integral part of problem solving, (iii) plays a key role in the study of other areas of mathematics for example fraction parts are related geometric part-to-whole constructs and ratio and proportion to geometric concept of similarity, and (iv) can provide a complete appreciation of the world and is enjoyable. All these geometric attributes can be attained in a classroom that allows freedom for learners to explore geometry and a curriculum design that integrates geometry with other sections in mathematics such as graphical representations related to the learner's day to day living.

Spatial sense is vital for study of formal geometry. Bruce, Moss & Ross (2012), Clements and Sarama (2011) note that in spite of its importance, geometry receives the least amount of time compared with other strands in classroom instruction. Spatial thinking allows mathematics to be become a more visual endeavour and connects with what is used by mathematicians to explore patterns in the world and make discoveries (Johnson, 2014). Research also shows that spatial skills might be predictive of later mathematics achievement (Drefs and D'Amour, 2014). For example, a recent longitudinal study with three-year-olds found evidence that spatial skills were even more important than early mathematics skills and vocabulary at predicting mathematics performance at the age of five (Farmer et al., 2013). Learners in the intermediate phase are expected to compare quantities and develop strategies to find perimeter, area together with dimension and symmetry from their visual interpretations. These should be enhanced by the development of spatial ability because research has shown that children with relative strength in mathematics have stronger visual-spatial abilities than verbal skills (Mix & Cheng 2012). Lohman, 1996, p. 98) defines spatial visualization as a specific type of spatial thinking that involves using ones' imagination to "generate, retain, retrieve, and transform well-structured visual images" sometimes known as thinking with the "mind's eye." The author further suggests that learners must be exposed to opportunities where they compose and decompose activities that allow them to visualise and imagine possible solutions before actually carrying out the task with manipulatives. The Geometry curriculum for the primary school should start with the real world of the child. The intuitive notions that children reveal when exposed to spatial situations should be capitalised on (van Hieles, 1982). Geometry starts when the child has to orientate him/herself in the everyday surroundings familiarizing with the physical environment. In theory a wide variety of examples and non-examples must be provided for learners to develop concept knowledge(Carpenter, 1986; Fuys & Liebov, 1997).In the intermediate phase learners could be given examples and non-examples related to comparison of quantities and development of strategies to find from their visual interpretations, perimeter, area together with dimension and symmetry. At this level the visual representations should be understood through the reflection of information related only to the course content.

Jones (2002) sees geometry as an integral part of our cultural experience being a vital component of numerous aspects of life from architecture to design in all its manifestations and that it appeals to our visual, aesthetic and intuitive senses. He also alludes to the fact that learners who find other areas of mathematics like algebra and number manipulations difficult to understand may find excitement, interest and creativity in geometry and be successful in mathematics. He therefore suggests that teaching geometry involves knowing how to recognise interesting geometrical problems and theorems, appreciating the history and cultural context of geometry, and understanding the many and varied uses to which geometry can be applied.

In order for learners to think about mathematical ideas there is a need to represent them internally in a way that allows the mind to operate on them. This argument contradicts Vinner's (1991), who identified mathematical definitions as playing a central role in the exploration, development and teaching of mathematics. To this Freudenthal (1971: p.67) pointed out that 'though the teacher can impose definitions…, this means degrading mathematics to something like spelling, ruled by arbitrary prescriptions.' de Villiers (1998) took up the issue of teaching geometry through definitions or teaching learners how to define in his study with secondary school learners. He concluded rather that learners need to be actively involved in formulating and evaluating definition of concepts. Nonetheless (Usiskin, 1996) recognizes the use of learning tools of both mathematics and ordinary language such as listening, writing, speaking, memorizing models and learning the history and culture. He therefore suggests that learners should master the use of the mathematics language as a tool that helps them with (i) the discovery of deeper mathematics, (ii) the heuristic exposition of complex mathematical ideas and (iii) the emotional experience of doing mathematics. Thus emphasizing the genetic approach to the teaching of geometry and advocating that learners be guided to a knowledge developing journey where the basic geometric concepts learnt in primary schools are developed to higher levels through knowledge of distinguishing object properties and relevant mathematical terminology.

The current approach to geometry in pre-schools to foundation phase curriculum includes little more than just recognizing and naming geometric shapes (Porter,1989). In the intermediate phase from grades 4-6 the curriculum tends to require naming of the geometric objects without engaging learners in deeper levels of classification and analysis (Fuys, Geddes and Tischler, 1988). Meanwhile the senior phase requires learners to be competent in solving simple geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties and definitions (DoE, 2011). Ironically (Porter, 1989, p.11) advocates that the fourth-and fifth-grade teachers across entire country spend "virtually not in teaching geometry". Also the current practices in the primary grades promote little conceptual understanding of geometric concepts. They do not allow learners to build mental constructions connecting their visual experiences with concept learning.

Jones, Fujita, and Kunimune, (2012) identify issues of mathematical definitions, mathematical representations, and the form of teacher's instructions in addressing geometry. They further believe that: a) definitions are essential to mathematical reasoning, b) in geometry lessons, geometrical shapes are represented in various ways and by various means (such as representing 3-D objects by 2-D representations), and c) the form of instruction used by teachers during their lessons is known to impact in various ways on students' reasoning (Jones et al., 2012). In this way, learners who find other areas of mathematics such as algebra and number manipulations difficult to understand may find excitement, interest and creativity in geometry and be successful in mathematics. In its summary of results and strategies for teachers in the primary division 2013-2014, (Education Quality and Accountability office, 2014) suggested that the teacher needs to have the learners describe using mathematical terminology what they see, consolidate that knowledge by using drawing illustrations and by adding it to classroom word walls. The teacher must provide learners with frequent opportunities to reference and use the relevant mathematical terminology. Although many physical objects have a geometrical shape which can be examined by the human senses (i.e. they can be touched, seen, constructed, and so on), they are also abstract mathematical objects. Thus the learners can connect the mathematical terminology to a visual image pasted on the wall.

Geometry is regarded by Lappan (1999) as a forgotten strand of mathematics. This is despite the fact that it offers us a way to interpret and reflect on our physical environment and can also serve as a tool for studying other topics in mathematics and sciences, it still has received little attention in instruction in schools. Jamison (2000) and Usiskin (1996) distinguish between three ways in which the use of language in mathematics differs from the language of ordinary speech, that (i) there is no present, past or future in mathematics, a concept 'is', (ii) it has no emotional content and (iii) it has no ambiguity. Hilton (1986) argues that mathematics cannot be learnt without being understood. He further asserts that mathematics should be acquired through a systematic thought. Systematic thought requires verbal expression. Geometry is usually communicated in the format of definition-theorem-proof. This implies that this communication should start with definitions. Definitions are concise statements of basic properties of a concept which unambiguously identifies that concept. For example: A square is a quadrilateral with all sides equal and angles are right angles. Indigenous language can be used to successfully create this meaning abstractly to learners such that they would be able to provide a suitable structure to this definition.

Piaget and Inhelder's (1967) theory on geometric thinking, learning and teaching included two themes that (i) representations of space are constructed through the progressive organization of the learner's motor and internal representations and that (ii) the progressive organization of geometric ideas follows a definite order that is more logical than historical. They believed that the representation of space is not a perceptual reading off of spatial environment but rather that the representation is built up from prior active manipulation of that environment. This implies that learners' ideas about shape do not come from passive looking, but are built as learners use their bodies, hands, eyes and mind engage in constructions and explore the attributes of the shapes. Contrary, our indigenous languages in South Africa lack equivalent terms that significantly explain mathematics and its manipulations. For example, '*a square and a rectangle*' share the same name, 'umboxo buxande' in Xhosa although they are different and there is no particular word for a '*diagonal*'.

Clement (2003) argues that cultural comparisons yield significant implications for the teaching of geometry. This was witnessed by the Japanese teaching which employed an inductive approach including the use of geometric representations problems in two or three dimensions which involved teaching practice that predicted high achievement. Also in the US, the Native black American high school learners demonstrated a deep command of transformational geometry as it was related to their language and cultural background (Giamati and Weyland, 1997). Matang (1998) asserts that it is the type of classroom practices employed by teachers of mathematics that contribute greatly towards learners' learning difficulties in mathematics though much of it is external to ordinary classroom teachers. He further suggests that (i) a significant feature of a mathematics learning environment is that both the teacher and learners build the mathematics together in developing special pride in learning activities facilitated by the spirit of free and open investigations, (ii) the learning climate in the classroom should provide an atmosphere of open communication between learners and their teacher through cooperation and collaboration during which the teacher is expected to encourage learners to ask questions at the same time accept variety of problems from learners and (iii) mathematics instructional material should be relevant to the learner's interests and needs allowing for learner experimentation. To this, (Özerem, 2012) asserts that studying geometry is an important component of learning mathematics because it allows students to analyze and interpret the world they live in as well as equip them with tools they can apply in other areas of mathematics. van Hieles (1999) aver that instructional activities that develop geometric thinking should begin with exploration, move toward more focused activities through which specific concepts are built and the related language is introduced, and end with consolidating activities that allow students to integrate what has been learnt with what was already known. The instructional activities should follow a sequence.

Conceptual Framework

This study is underpinned by the van Hieles' Theory of Geometric thinking. This is one theory that offers a model for explaining and describing how learners think as they engage with geometry problems. Van Hiele (1986) advocates that the lack of prerequisite understanding about geometry creates a gap between learners' level of geometric thinking in which they operate, and the level of geometric thinking that they required for and are expected to learn. Van Hieles theory defines five levels of learning geometry which learners must go through in order to obtain an understanding of a geometric concept. These five levels are very valuable in designing activities and instructional phases although they are not strictly sequential (Usiskin, 1982). Table1 outlines the levels of geometric thinking.

Much of the research (Usiskin, 1982, Senk, 1989, Burger & Shaughnessy, 1986, De Villiers & Njisane, 1987) has confirmed that according to van Hieles' theory the levels are hierarchical, and can be used to describe the geometric thinking of learners and that each phase involves a higher level of thinking.

Table 1: Levels of geometric thinking (adapted from Fuys et al. (1988) and Presmeg (1991))

(a) **Inquiry/Information**	The learner explores and examines examples and non-examples of a certain structure by using the material presented to him/her
(b) **Directed orientation**	The learner explores the field of investigation using the material, either by folding, measuring, or looking for symmetry
(c) **Explanation,/explicitation**	The learner becomes conscious of the network of relations, tries to express them in words and learns the required technical language for the subject matter, for example, expresses ideas about the properties of figures.

(d) **Free orientation**	The learner is given more complex tasks to find his/her way round this field, for example, a learner might know about the properties of one kind of shape but is required to investigate the properties for a new shape, for example, a trapezium. The tasks should be designed so that they can be carried out in different ways.
(e) **Integration**	The learner summarizes all that she/he has learned about the subject, reflects on his/her actions and thus obtains an overview of the whole network/field that has been explored, for example, summarizes properties of a figure.

Van Hieles' (1986) saw learning of geometry as a discontinuous process characterized by qualitatively different levels of thinking. They described these levels as (i) visual level where learners recognize figures as wholes, (ii) description of properties of geometric figures, (iii) ordering and analyzing of geometric figures, (iv) deduction and abstraction of information from the figures and (v) rigor in proofs (Hoffer, 1981). The van Hieles also believed that these levels were sequential, invariant and hierarchical with instruction determining progress from one stage to the other and not age of the child. They further indicated that each level is characterized by its own language and thinking. Teachers have to be aware of this hierarchy of geometric language to ensure learners' understanding of geometric ideas. Although this theory does not outline detailed descriptions of children's thinking and representations of geometric concepts, it provides the general framework for curriculum and teaching.

Problem Statement

In South Africa, learners in the foundation phase, grades 1-3 receive their mathematics instruction in their mother tongue. However this changes in the intermediate phase and they learn mathematics in the universal language for the first time. This includes communication, the explanations on definitions and basic properties of geometric structures and concepts. For these learners to learn and understand how and why geometric figures differ from each other, they need to be conscious of the network of relations between the figures, express them in words and learn the required technical language to express ideas about the properties of geometric figures. This paper sought to establish how these learners construct their geometric meaning in order to understand and interpret geometric concepts.

Research Questions

This paper specifically responds to the following questions:

- How do learners construct their geometric meanings for the understanding of geometric concepts?
- What were the learners' difficulties in the interpretation of geometric concepts in the intermediate phase?

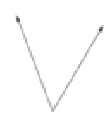
Methodology

This paper reports specifically on the main study data collected for the Mathematics Teacher Learner Intervention Programme (MTLIP) aimed at identifying how learners interpret geometric concepts in the intermediate phase. This study is located within the interpretative research paradigm. This was an exploratory research in which qualitative methods which emphasize the use of test questions and probes, give participants an opportunity to respond in their own words (Devetak, Glazar & Vogrinc, 2010) were used. The exploratory design helped to determine the current level at which learners' geometric thinking could be classified. This study was also quantitative in that it was conducted with 219 grade 5 learners from 20 randomly selected primary schools in the Limpopo province of South Africa. Initially, in September 2013, a pilot study was conducted with 183 grade 4 learners from 10 randomly selected primary schools in the Mamelodi district of the Gauteng province in South Africa.

Instruments

From the pilot study, statistical analysis of the results to measure content validity indicated a 1,1% pass percentage of learners' poor performance reflecting poor learner content knowledge. The learners' problems associated with learner content knowledge were found to include (i) misunderstanding of the questions, (ii) learner poor background in the language of instruction knowledge, (iii) the structuring of the items in the instrument, (iv) learners' inability to follow the instructions needed to answer the questions, (v) misrepresentations of response items from the learners, and (vi) learners' inability to answer questions on areas, identification and naming of angles. For the reliability of the instrument various questions on the questionnaire were revised and replaced with new ones. For example, question 9 in the questionnaire was initially posed as: The internal angle in the figure is:

A bigger than 90^0
B 90^0



C smaller than 90^0
D triangle

The poor performance in the pilot study could also be attributed to some content that had not been covered during the period of administration of the questionnaire. Hence after the revision of the questionnaire, the researcher decided to administer the questionnaire to grade 5 learners at the beginning of the 2014 since they would have covered all grade 4 content as prescribed by CAPS for the main study. This was the appropriate time for grade 5's since those concepts would have been fresh in their minds and would serve as the prerequisite knowledge for mathematics concepts to be learnt in grade 5. The questionnaire covered the whole grade 4 syllabus, but this paper reports only on the understanding displayed in responses to five out of fifteen questions that sought interpretation of geometric concepts in the instrument. All questions in the instrument were presented in a multiple choice format, each with four options from which learners had to choose the suitable answer. Questions on Geometry were covered in questions 8, 9, 10, 14 and 15.The schools were randomly selected primary schools in four circuits of the Limpopo province wherein five schools were chosen in each circuit. The schools were then coded per circuit starting from A1; A2… to D5. A total number of 219 learners wrote the test.

The grade 5 learners responded to the questionnaire in the form of pen and paper test conducted for forty minutes. The questionnaires were then marked to establish the correct and wrong responses. Unstructured and semi-structured interviews were then conducted with six learners on the basis of their responses in the questionnaire to gain more insight into how they constructed their geometric meanings and interpretations. A discussion of the types of structures constructed by learners when learning the mathematical concepts with the view to(i) clarify their understanding of geometric concepts, (ii)find out how the lack/availability of these structures hamper or assist learners' understanding of geometry, (iii)determine the learners' actual engagement with tasks and how these tasks link with the expected outcomes highlighted in the CAPS document, and (iv) inform possible modifications to enhance learners' understanding of geometric concepts. This was done to check whether they had a coherent understanding of geometry.

Limitations

The questionnaires were administered to a sample of learners in rural schools of one province in South Africa. Although results obtained may be generalized for most learners of this phase in rural settings in the country, they may be different internationally.

Discussions and Findings

After marking the test from the administered questionnaires, follow-up semi-structured and unstructured interviews were conducted with six subjects based on their responses to the different activities in the questionnaire. The objective of the interviews was to (i) get clarity on the written responses,(ii)classify the level of geometric thinking and interpretation to which the learners operated. It was requested that some learners who struggled with explanations use their mother tongue during the interview to justify their understanding of geometric concepts and how they saw and interpreted the given diagrams. In these interviews the learners were asked to respond in an open-ended fashion to the following issues: (i) justifying the responses to particular questions in the research instrument, (ii) identification of concrete shapes brought along by the researcher, (iii) explanations on why they made such classifications.

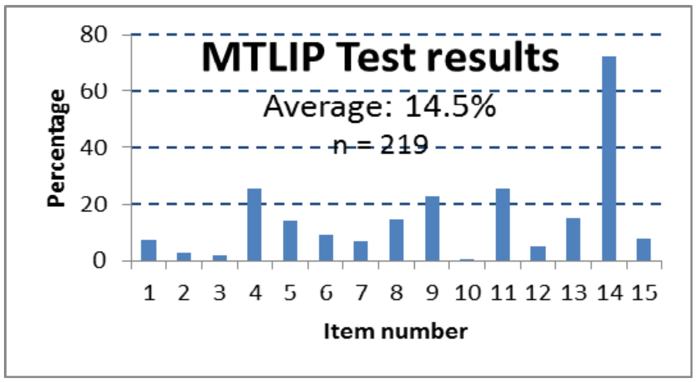
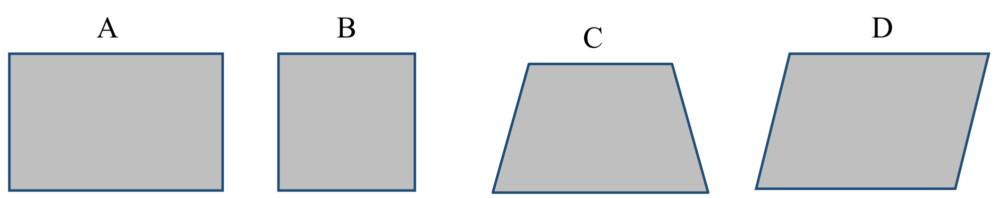


Figure 1: MTLIP Test results

On quantitative analysis of the test results an average 0f 14.5% correct responses was registered while reflecting a standard deviation of 1.452789. On average this reflected a very low and poor performance on the entire test implicating that the learners hardly understood the geometry content. The results will be presented in the form of (i) statement of question, (ii) results from data collected (iii) some interview transcripts and (iv) implications for the teaching of geometry.

A. Question 8

Which shape is a square? Draw a circle around the correct letter.



In this question, only 14% responses indicated the correct response. About 56% of responses indicated A as a preferred answer. When one of the learners was interviewed on why she chose A as the correct answer, she said:

Learner's statement: *Ke yona ntho thisha are rutang yona,* (This is what our teacher taught us).

Probing further, the researcher asked the learner to pick up a square shape from four concrete shapes that she brought along. Two of those were squares in different colors, one was a rectangle and the last one was a triangle. The learner chose both square shapes. Asked why or how different were they from the other shapes, she indicated that she could see that they were squares. She could not cite any of the properties that make the shape to be a square. The level of understanding for this learner only operated as far as visual level where learners recognize figures as wholes. Some learners also mixed the properties of a square by indicating that the rhombus was a square since all of its sides are equal. Such learners can't separate examples from non-examples. It could help if learners are exposed to investigative questions in order for them to establish why a square is a square. What critical features make this object a square? In this way their level of interpretations can show some construction of geometric meaning.

From the Figure 1, it can be seen that although the average achievement performance in all fifteen items was 14.5%, only 22% of responses were correct for item 8. This indicates that most learners experienced challenges on identification of properties of the square.

Question 9

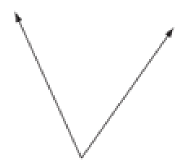
The given angle is:

A bigger than a right angle

B a right angle

C smaller than a right angle

D a triangle



Draw a circle around the correct letter

About 22% of learners indicated a correct response to this question. Probed further on why they chose option C,

Learner said: *I knew that it was not a triangle because it was not closed, so I just chose the next option above.*

Researcher*: Do you know a right angle?*

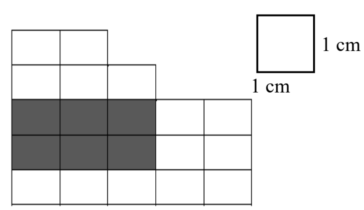
Learner*: Maybe the angle on the right? I am now confused*

This clearly indicates that the learner had no idea of a right angle but used guess work and landed with the correct answer. Meanwhile 54% of the responses chose the ‘triangle’ option. Although in the representation no indication of triangle was given, triangle was the most familiar word for them in the given options. Most of the learners just completed the ‘Given’ structure by drawing a line to make a closed figure. The other explanation would be an interpretation from African languages that were used in the learners’ previous grades. One learner tried to make a fold in the middle of the given figure and ended up saying that this should be a triangle. Learners showed difficulty with measurement since they had no clue of what was meant by a right angle.

Question 10

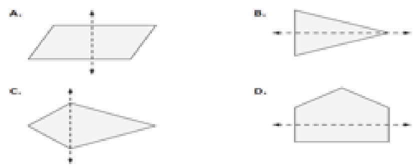
What is the area of the shaded part of this diagram?

Your answer:



‘*The area is black’ ‘It’s a rectangle*’ ‘*The area is the one that has been colored’* were some of the responses to this question, indicating that the learners did not understand the question. Most learners who gave correct responses to this question just wrote 6, but not 6cm^2. One learner wrote 6/14 or 6/20 where he/she did not understand the question on areas but is used to such diagrams shaded as fractions of the whole structure. He/she saw the surface of what the question should look like rather than a deep sense of trying to understand the question. Some learners interpreted the shaded area as a square and others just writing ‘diagram’. Most learners operated in a level much lower than even the inquiry level as they did not understand what the question required. Learners experienced difficulties in understanding the mathematical interpretation of the word ‘Area’, they had no clue on how to make calculations and what is expected of them.

B. Question 14 Which one of the following figures shows a line of symmetry? Draw a circle around the correct letter.

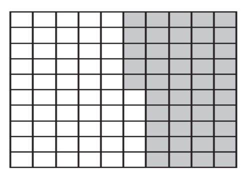


This question enjoyed more than 90% correct responses from all the schools. This one learner attributed to the practical examples illustrated for them when this concept was introduced. These included paper folding with exact edges lying on each other. It appeared as if most learners had a schema of the symmetry concept. With this concept the learners went beyond direct orientation in their geometric thinking, and were aware of examples and non-examples of the concept of symmetry. Explanations were not conscious of the network of relations though, and could not express these relations in words because of lack of the required technical language for the concept of symmetry nor could they express ideas about the properties of the given figures. This indicated that they had shallow knowledge about most geometric concepts.

C. Question 15

The shaded area in the figure shows a fraction. What fraction of the figure is shaded?

Your answer:



Most learners gave 45 or 45/55 where the learners have counted the shaded ones in relation to the non-shaded ones. The idea of a whole is not well developed. Learners looked at the fractions as whole numbers because many of them wrote 45 instead of 45/100. It seems with fractions that teachers might have moved to symbolism very quickly and as such only 8% learners gave a correct response for this question. Language versus diagram representations, contradict each other, for example in this question one of the responses indicated that 'the shaded portion was on the right'. Diagrams and pictures are always used by teachers to facilitate understanding but some responses indicated the deviation of the mathematics meaning from an anticipated response by misleading them due to language problems. They mostly did not concentrate on what the question looked for. When a question has words and a diagram, learners tend to ignore the words or diagram and concentrate on one of those representations.

Conclusions and Recommendations

The results in this study indicate a classroom situation, where the teacher, the texts, and the learners are functioning at different levels and hence using different linguistic symbols or networks of relations. Consequently, the learners and the teacher do not understand each other (Mason, 1998, and van Hiele, 1986). (Oyoo, 2009, and van Hieles, 1986) refer to this scenario as failure of the teacher to deliver teaching to the learners in a language that is terminologically appropriate to the learners' thinking levels. This study revealed that it was difficult for learners to show evidence of geometry knowledge construction since theywere obliged to imitate, but without understanding, the action structure of the teacher. It is therefore recommended that a teacher beginning the teaching of geometry should address himself to the learners in a language familiar to the learners so that learners can understand geometric concepts. Teachers should use level-appropriate terminology, symbols, and general language in their geometry teaching practices. A gradual introduction of geometric terminology associated with concrete object manipulation can be effective in building meaning and understanding of geometric concepts.

Learners could also not interpret diagrams. The inclusion of a diagram in some questions tended to mislead the learners' understanding of the question. The development of interpretation skills for the diagrams was not adequate. The learners try to get geometric meaning from the given diagram but the language demands render the learner at a disadvantage. It is proposed that the introduction of concepts from pictorial representations to symbolic abstractions should be carried out at a very slow pace. It was observed in this study that learners just pick up just one word and try and associate that word without making sense of the whole text in the question. There also appears to be a problem with the meaning of the word, 'shaded'. The problem was that learners seemed not to comprehend the questions in the questionnaire. Perhaps if the question was asking for the area of the black space, some mental constructs related to counting squares covered on the black area would have helped learners to make sense of the question. Thus learners could not construct clear geometric meaning due to lack of understanding of the geometric language used in the questionnaires. Their intuitions were only built on what they saw but could not reconcile with relations on geometric concepts.

It is recommended that learners at this level be given word problems together with number sentences that vary in difficulty. Also visual learning can be enhanced by introducing geometric concepts through allowing learners to investigate various features that distinguish objects from others. As the learners grapple with those facts they will acquire information through illustrations through drawing of well-chosen contexts. The explanations on how they compare and contrast the features of available objects form the basis of their investigations, develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. The geometric language should therefore be enhanced from the mother tongue to the language of instruction used in the intermediate phase. The teacher can use models like photos, diagrams, graphs, symbols, icons, and other visual representations. It is also suggested that mathematical concepts such as comparison, scale, dimension, measurement, direction, shapes, and perspective, symmetry should first be experienced visually before they can develop to exploration of figures abstractly. This can be accomplished when the teachers engage the learners in reflection and articulation of their reasoning by use of probing questions, 'why do you do this and how did you get that. When learners are requested to share their thinking about how they worked on a problem, grapple with facts and evoke critical thinking often helps them identify their own mistakes or the flaws in their reasoning. They would therefore have no difficulties in interpreting meaning on geometric concepts.

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