Algorithm for an Efficient Material Requirements Planning of Consumable Supplies Results of an Empirical Analysis of German Supply Data

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ABSTRACT

The present paper aims at describing a standardized algorithm, with which the consumption of tie-in products can be forecasted with different quantitative forecasting techniques. Among the techniques are AR-, MA-, ARMA-, ARIMA- and structural regression models. By means of algorithmic procedures, an optimal forecasting model of performance criteria will be evaluated, which will be used for requirements planning. For all determined forecasting horizons, the ARIMA-model proved to be the best model followed by simple Moving Average models. The importance of autoregressive procedures decreases with the length of the forecasting horizon. Structural approaches seldom prove to be the best forecasting models, even if the importance of these increase according to the length of the forecasting period. The algorithm allows for a considerable part of tie-in products a good forecasting quality. Considering elaborated premises the algorithmic procedure simplifies the forecast of the outflow of goods for a considerable part of consumable supplies.

INTRODUCTION

Materials Requirements Planning in Inventory Management of a German Mechanical Engineering Company

According to economic definition, stocks are knots in the logistic system, in which goods are held temporarily and changed in their composition of quantity. Stocks have the function of supply and receiving points, as well as of disintegration and concentration points. There are receiving stocks, interim storage facilities, product stocks, manufacturing stocks as well as dispatch stocks. The task of every inventory management policy consists of effectuating a stock-keeping at optimal cost depending on input and output flows. Stock- keeping

systems with fast reactions become necessary (Schäfer-Kunz/Tewald 1998). Input flows in one- (for classifications cp. (Schneeweiß, 1981: 45-48) and (Porteus, 1990: 605-608) and multi-period models (for the topic of supply-chain-management cp. e.g. (Bowersox/Closs, 1996: 88) and (Pfohl, 1996: 175) of inventory management can be directed through order size and order point of time, output flows of demand for finished products are directed exogenously. Based on this fact theoretic inventory management models with deterministic demand are applicable only in exceptional cases. These models do not fulfil the assumption of random stock flows. However (Hillier and Liebermann, 2001: 987) state that deterministic models are a sufficient approach to a lot of inventory management situations in practice: "For example the Economic Order Quantity (EOQ) models have been particularly widely used. These models are sometimes modified to include

some type of stochastic demand, such as the stochastic continuous reviewed model does". Basically, the simple and intuitive models of inventory management are applied in practice. This is true especially for B and C-classified products of an ABC-analysis, because a systematic application of complex procedures for the multitude of products of a company overstrain small and medium-sized companies but also big and globally operating ones. Especially mechanical engineering companies are faced with this problem, where the amount of applied products is disproportionally high. Therefore, the inventory management often relies on "hand-knitted" or standard software-based solutions (for the use of ERPs in small and medium-sized enterprises see e.g. (Van Everdingen et al., 2000: 27-31) as well as e.g. (Davenport, 2000) for the implementation of ERPs, see (Razi and Tharn, 2003) for inventory management in ERP systems), where the experience and intuition of the inventory manager determine the order size and point of time. Very often the real and potential receipts of orders of the sales department are used as possible predictors. This is a very unsatisfying solution because sales departments by trend overestimate their "initiations" because they rate their own ability to deliver on time higher than the costs of an unnecessarily increased capital commitment in inventory. However, holding inventory is very costly for companies who in fact feel the need to reduce inventory costs (Masters, 1993).

In the past, this led many purchase departments of medium-sized companies to consider to what extent the stochastic demand for feedstock – that is the commodity flow out of the stock - can be forecasted in view of the optimal order quantity and the optimal order point of time for the receiving store without having to bind personnel and financial capacity in the company. Models to forecast product demands have been analysed under different special conditions like normal distributed lead

times (Chopra and Reinhardt/Dada, 2004) and poisson-distributed demand (Hollier, Mak and Lai, 2002). (Lindsey and Pavur, 2008) and (Lindsey and Pavur, 2009) predicted future demand rates of products with intermittent demand, i.e. with observed demands of zero. The inventory costs should be reduced as much as possible. For this purpose, a reliable forecast is important. A standardized solution to forecast demands of products which however involves the characteristics of the product would be desirable. The present paper describes a standardized algorithm with which the consumption of 110 tie-in products of a German medium-sized mechanical engineering company can be forecasted for a period of three, six and twelve months supported by econometric procedures. In the scope of the developed algorithm, different quantitative forecasting techniques are applied for each product group and evaluated in regard to different criteria of forecasting quality. The main goal of the article is not to present the different-already well known - quantitative methods but to present how their systematic algorithmic combination can provide some new insights into the real-life use of quantitative forecasting methods for companies in the mechanical engineering industry. The developed approach should enable companies to use also more complicated quantitative methods for forecasting with relatively little effort. This systematic application allows a good predictability for a considerable part of tie-in products.

Applied Forecasting Techniques for Material Requirements Planning

The consumption of N=110 tie-in products exists on a monthly basis for a range of N=77 months. For the forecasting of the product time series $\{z_t\}_{t \in T}$ we choose ten different econometric models which will be differentiated at first, so that the requirement is to be

seen in rates of change. Normally, this leads to stationary data series – a prerequisite for the application of econometric time series methods. In order to prove the stationarity of the time series, the Augmented Dickey Fuller Test (Dickey/Fuller 1979) will be conducted. If there is no stationarity of time series, then successive differences up to the third order will be created. If the time series in the third order still is not stationary we adapt the models to the first difference, whereas a warning of non-stationarity is given. The applied difference is referred to as d, the d-times differentiated time series as $\{y_t\}_{t \in T} = \{\Delta^d z_t\}_{t \in T}$.

On the basis of this time series different econometric forecasting techniques will be applied, which can be divided into two forecasting groups: (1) time series methods and (2) causal methods. The time series methods are limited to the use of patterns and structures of historic data of a variable, in order to extrapolate it into the future. For the forecasting of outflow of goods, only information about the outflow of the same product in the past will be used. The decision which and how much information of the past shall be applied will be carried out with a minimization of the Bayes Information Criterion (BIC) (Akaike, 1977). This criterion determines the ideal lag-structure of the product variables. The econometric times series forecasting techniques are described in figure one as models one to seven. After setting up the models, a Portmanteau-test for the residuals will be carried out (Box and Pierce 1970; Ljung and Box 1978). The amount of collective autocorrelations of the residuals tested for zero was chosen as 20% of the used observations.

Structural forecasting techniques assume that for the development of a variable other explanatory effects or variables are responsible. These explanatory variables must have the quality to run ahead sufficiently in time of

the development of the product flow, in order to predict the product flow at the point of time t at an earlier time point t-x. A series of predictors can be considered as explanatory variables, which in practice are used as leading indicators of economic development. Whereas in the USA, the Purchasing Manager Index (PMI) is used (Niemira, Zukowski, 1998; Lindsay and Pavur, 2005) for the new IPMI, in Germany the "German Association Materials Management Purchasing and Logistics e.V." (Bundesverband Materialwirtschaft, Einkauf und Logistik e.V.) and the Ifo-economic climate index (Kunkel, 2003) count for the developments of the German market as leading indicators. These indizes - available for everybody - are secondary data sources which reflect the estimated economic development of the German market. Especially for the latter mentioned index, branch specific individual results are being published for the total index as well as for single partial indices.[1] If the consumer industry of a company rates the business situation as regressive, then the sales volume of the supplier industry will decline in the near future. As predictors for the flow out of a purchasing stock, the eleven partial indices of the respective consumer industries are theoretically suitable. In the structural forecasting models (8 and 9) in figure 1, it is assumed that the demand time series of the chosen product depends linearly on the mentioned eleven economic indicators. The tenth forecasting method tries to unite the results of the time series method 4 with the structural forecast method 8.

[1]See footnote in figure 1

Figure 1: **Applied forecasting techniques**

	Model	Description
Reference model	1	A Random Walk $y_t = y_{t-1} + \varepsilon_t$ will be used as a **naive comparability and reference model** for the following procedures. The forecast for the random walk thus yields $\hat{y}_t = y_{t-1}$. The forecasted value thus corresponds with the value of the last observation.
Time series methods	2	**MA(q)-Model**: A natural Moving Average Process of order q^2 will be fitted: $y_t = \varepsilon_t - \beta_1 \varepsilon_{t-1} - ... - \beta_q \varepsilon_{t-q}$
	3	**AR(p)-Model**: A natural autoregressive process of order p will be fitted: $y_t = \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + \varepsilon_t$
	4	The optimal order of an ARMA(p,q)-model $y_t = \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} = \varepsilon_t - \beta_1 \varepsilon_{t-1} - ... - \beta_q \varepsilon_{t-q}$ is determined on the basis of the Hannan-Rissanen (Hannan, Rissanen, 1982) procedure. To begin with an AR(p) model is assumed. Order p, determined as the optimal order in model 3, is chosen for the adjustment of the process. For the adjusted process, the residuals and the lags of the residuals will be determined. This is followed by a regression of the residuals' lags and the autoregressive terms calculated as optimal onto the independent product variables. The optimal lag-structures to be used will be determined by the Bayes Information Criterion (BIC). As the ARMA-process has been adjusted for the d-times differentiated time series, this gives an ARIMA**(p,d,q)** model for the time series $\{z_t\}$.
	5	The values of the time series of the observed product will be logarithmised to begin with, that is the time series $\{\ln(z_i)\}_{i \in I}$ will be considered. For the transformed time series, an optimal ARIMA-model – like in model 4- will be determined in dependence on the Hannan-Rissanen procedure. The order for the difference d corresponds again with the prerequisites from the Augmented Dickey Fuller Test. For the differentiated logarithmised time series an optimal AR(p) process will be adjusted, for which the residuals and the residuals' lags will be calculated. The potential AR- and MA-terms will be regressed onto the time series. Orders, for which the Bayes Information Criterion is minimal, will be chosen.
	6	Furthermore, the demand time series of the examined product will be smoothed out over a period of 3 months. Here, the last observed value will be weighted a half, the penultimate and antepenultimate will be weighted each a quarter, so that $z_{t,new} = 0,5 \cdot z_t + 0,25 \cdot z_{t-1} + 0,25 \cdot z_{t-2}$. To the resulting time series an ARIMA(p,d,q) model will be adjusted according to the Hannan-Rissanen procedure.
	7	In model 7 a level component is considered for the adjustment of the time series, that is for the **ARIMA(p,d,q)** process a **continuous term** is allowed, so that $y_t = \mu + \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} = \varepsilon_t - \beta_1 \varepsilon_{t-1} - ... - \beta_q \varepsilon_{t-q}$.

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[2] For an overview on time series analysis see e.g. Box and Jenkins (1970), Hamilton (1994) or Stier (2001).

Structural Regression	8	Here, for each individual independent variable the optimal lag-lenghts will be calculated given a multiple regression onto the product variables. It is achieved by testing different successive lag-lengths and combinations of different lag-lengths as regressors. With the help of the Bayes Information Criterion (BIC) the optimal lag-structure is determined. For the optimal lag-structures of all independent variables calculated in this way, a step by step backward regression onto the dependent variable is carried out in a second step. In each cycle, the variable with the highest p-value is excluded of all variables that show a p-value greater than 0.05. This will be done until only significant regressors (p≤0.05) remain in the model.
	9	This model corresponds to model 8, for which a forward regression instead of a backward regression is carried out with a step by step inclusion of variables with the smallest p-value smaller than 0.05. If an independent variable that is already in the model loses its significance during the running iteration process, the variable will be excluded from the model if its p-value is greater than 0.1. The potential independent variables are those that were also determined for model 8.
Mixed Procedures	10	Finally, models (4 and 8) will be combined. For the product variable $y_t = z_t - z_{t-1}$ a regression onto the first differences of the forecasted values from the ARIMA(p,d,q) process in model 4 $(f_{4,t} - f_{4,t-1})$ and the structural model 8 $(f_{8,t} - f_{8,t-1})$ will be effectuated. The forecast for y_t results from adding the estimate of the regression function to the already known value, that is $y_t = y_{t-1} + \hat{\beta}_0 + \hat{\beta}_1(f_{4,t} - f_{4,t-1}) + \hat{\beta}_2(f_{8,t} - f_{8,t-1}).$

The part indices are (1) current business situation, (2) production compared with previous month, (3) assessment of stocks of finished goods, (4) demand compared with previous month, (5) domestic sales prices, (6) orders in hand, (7) orders from abroad, (8) export expected for the next three months, (9) development of the number of employees, (10) selling prices for the next 3 months, (11) business expectations for the next six months.

Measures Used for The Evaluation of Forecasting Quality

In order to make sure that the forecast can be generalized, the cross-validation-method (Devijver and Kittler, 1982; Geisser, 1993) is often applied in practice, which divides the data set into two parts. On the basis of the so called training data set the forecasting model is developed, in order to examine it for its quality on the most recent data of the time series - the so called test case.

For the present data, a test data set was cut for the time series for the period of forecasting in the following way. For the short-term forecast over a three-month period, a training data set ranging over the entire time series could be applied minus the last three months. The three-month test case on the current edge serves as a test for the forecasting quality. The longest demand forecasts extend over 12 months, so that the test case in this extreme case still showed (77-12=) 65 observations.

In the following, several different measures were computed and applied to the training data set and the test data set, in order to evaluate the forecasting quality.

1. The "Root Mean Squared Error" (RMSE)[3] is the measure most often applied for forecasting evaluation. The mean square deviations of the forecasting values are averaged from the real values. Taking the squared deviations, larger deviations are weighted to a larger extent.[4] The RMSE results from drawing the root of the MSE, that is

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (y_t - \hat{y}_t)^2}$$

2. For the standardization of the forecasting error the forecasting measure must have a reference value. Theil`s inequality coefficient[5] relates the root of the mean square error (RMSE) for the forecasting model to the RMSE of the naïve forecast, i.e.

$$U = \frac{\sqrt{\frac{1}{N} \sum_{t=1}^{N} (y_t - \hat{y}_t)^2}}{\sqrt{\frac{1}{N} \sum_{t=1}^{N} (y_t - \hat{y}_{reference,t})^2}}.$$

In case of a perfect forecast, the MSE and thus Theil`s inequality coefficient take on a value of zero. If the forecast is only as good as the naïve forecast of an extrapolation of the respective last observation into the future, then Theil`s inequality coefficient has a value of one. If Theil`s inequality coefficient takes on values greater (smaller) than one, then the chosen forecasting procedure is worse (better) than a naïve extrapolation of the previous values into the future. Admittedly, a material requirements planning would hardly extrapolate only previous values. Instead, values of a "common for business" forecast as series of reference would have been adequate. However, these were not available, so that at this point it will be calculated with the

usual series of reference of the naïve extrapolation of previous values.

3. The mean per cent deviation MPA[6] averages the per cent deviation of the forecasts in respect to the real data, that is

$$MPA = \frac{1}{N} \sum_{t=1}^{N} \frac{|y_t - \hat{y}_t|}{y_t}.$$

4. For the practical procurement forecast another measure is of importance. It is often only of interest whether the needed order quantity in its total amount is well forecasted for the entire forecasting period and not over- or underestimated. Upward or downward deviations can be balanced out by trend in the forecasting period, if the error term is not autocorrelated. Accordingly, for the average total per cent deviation in the forecast period (MGPA)[7] the sum of the forecast errors is related to the sum of the real demand values, that is

$$MGPA = \frac{\sum_{t=1}^{N} (\hat{y}_t - y_t)}{\sum_{t=1}^{N} y_t}.$$

In procurement management, it is not only important to determine a point estimate with the lowest possible forecasting error for the future demand of products. It is rather of interest whether the potential forecasting errors are symmetrically spread and without great outliers around the point estimates. Especially outliers can lead to massive over- or underestimation concerning future demand, which in the end can lead to high capital lockup or loss of output.

Therefore, the skewness and the kurtosis of the forecasting errors were determined for the test data set. The test of skewness and kurtosis was conducted

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[3] Compare e.g. Khosravie-Rad (1991), pp. 223-224; Schwarze (1980), p. 329; Masters (1998), p. 281.
[4] See Theil et al. (1971), p. 124.
[5] For an overview on Theil's inequality indices see e.g. Hujer/Cremer (1978).

[6] See Schwarze (1980), p. 329 and Brooks (1997), p. 134..
[7] Cp. Khosravi-Rad (1991), p.222.

according to D`Agostino, Balanger and D`Agostino Jr. (1990) with the empirical adjustment of Royston (1991). A right-skewed distribution of forecasting errors $e_t = y_t - \hat{y}_t$ that is the intensified appearance of negative forecasting errors indicates an above average overestimation of individual values, a left-skewed distribution an above average underestimation. With a p-value less than or equal to 0.05 and skewness greater than zero, the distribution will be assumed to be right-skewed, with a negative skewness the distribution will be assumed to be left-skewed. With an insignificant p-value greater than 0.05 there are no uneven extensive over- or underestimations and we call the distribution `not skewed`. If the value of the kurtosis is greater than 3 with a p-value less than or equal to 0.05, we can assume that the distribution tends to be `strong outliers`, with a kurtosis less than three and/or an insignificant p-value greater than 0.05 `the distribution does not tend to have strong outliers`.

Forecasting Power of the Algorithm

The N=110 data series were tested with the above described ten forecasting techniques in short-, medium-, and long-term forecasts. For the short-term forecasts, a time period of three months was determined, for the medium-term forecast six months and for the long-term forecast twelve months. The forecasting power of these was evaluated separately from each other in the course of time.

For the adaptation of the models a model was chosen as optimal, which showed the least Theil`s inequality coefficient during the test phase and whose residuals could be regarded as `White Noise`. The latter precondition is being tested with the Portmanteau Test. If the Portmanteau Test is negative for the automatically chosen model, then the model with the lowest Theil`s inequality coefficient is used in the training phase, for which the Portmanteau Test shows that the residuals are independent and standard normally distributed. The above described quality characteristics of the forecast in regard to the training and test data set therefore allow further statements concerning the quality of the chosen forecasting model.

In total, it can be seen that the adjusted models in the training data set strongly tend more to the above-average strong overestimation than to the above-average strong underestimation of the actual values. A strong overestimation of the actual values appears in 39.8% to 44.4% of the cases depending on the duration of the time period. Combined with the fact that more than 50 % of the time series tend toward outliers (see kurtosis in figure 1) this is a first indication for the fact that a considerable amount of product series is characterized by sporadic slumps in demand and even by `zero-demand` periods. Sporadic outliers upwards appear seldom. Only 2.7% to 4.6% of the forecasts show a significant underestimation (cf. figure 1)

Figure 1: Skewness and kurtosis of forecasts with optimal predictive power in the training data set

Skewness	Short-term forecast (3-months forecast)		Medium-term forecast (6-months forecast)		Long-term forecast (12-months forecast)	
	Quantity	Per cent	Quantity	Per cent	Quantity	Per cent
Right skewed	3	2.7%	4	3.7%	5	4.6%
Not skewed	62	56.3%	61	56.5%	55	51%
Left skewed	45	40.9%	43	39.8%	48	44.4%
Total	110	100,0%	108	100,0%	108	100,0%
Kurtosis	Short-term forecast (3-months forecast		Medium-term forecast (6-months forecast)		Long-term forecast (12-months forecast)	
	Quantity	Per cent	Quantity	Per cent	Quantity	Per cent
Does not tend to outliers	53	48.2%	54	50,0%	53	49.1%
Tends to outliers	57	51.8%	54	50,0%	55	50.9%
Total	110	100.0%	108	100.0%	108	100.0%

Short-term Forecast

Figure 2 shows the distribution of the models chosen as optimal of the N=110 products for the short-term forecast of 3 months.

Figure 2:Models with best predictive power from the Theil`s inequality coefficient in the training data set (3 months forecast)

Type of forecast	Forecasting model	Chosen[1]	Worse than reference model		Better than reference model	
			Number	Per cent	Number	Per cent
Time series econometric procedure	Model 2	21.8%	5	20.8%	19	79.2%
	Model 3	5.5%	4	66.7%	2	33.3%
	Model 4	45.5%	18	39.1%	28	60.9%
	Model 5	1.8%	1	50.0%	1	50.0%
	Model 6	3.6%	2	50.0%	2	50.0%
	Model 7	19.1%	7	33.3%	14	66.6%
Structural regression	Model 8	1.8%	1	50.0%	1	50.0%
	Model 9	0.9%	0	0.0%	0	0.0%
Mixed procedure	Model 10	4.5%	4	80.0%	1	20.0%
	Total		42	38.2%	68	61.8%

[1] Due to multiple nominations a total sum of more than 110 time series and a per cent sum of more than 100% resulted.

Model 4 (ARIMA(p,d,q) without level) was chosen in most cases that is in 45.5% of the cases, followed by model 2 (a pure MA-process) with 21.8% of the cases. In four cases, model 3 and model 4 led to the same model (pure AR(p)-model), in one case model 8 and model 9. In the following we assume 110 time series again, the identical results will be mentioned at the respective lower model number (that is for model 3 and model 8).

Figure 2 shows whether the results are similar regarding the application of the respective forecasting model to the test data set. For 61.8% of the models, Theil`s inequality coefficient is less than one also in the forecasting phase, that is the forecast is better than the results of the naïve model. Here the pure MA(q) model 2 with 79.2% is the best, followed by the ARIMA(p,d,q)-model with level (model 7) with 66.7%, as well as the ARIMA(p,d,q)-model without level (model 4) with 60.9%.

In total, the results are quite sobering because after all it was expected that by far more than 61.8% of the time series there would be better forecasts, than only through the naïve time series "update". It is thus not astonishing that only for 38.2% of the time series there is a medium per cent deviation of less than 20%. For about 32%, this deviation lies between 20 and 40 % and for about 30% of the time series it is even more than 40%. If you do not refer the error to the average monthly deviation but to the average deviation, which appears in the total forecasting time period of three months through aggregation of the forecasted and real monthly values to the quarterly values, the results are only slightly better. In 54% of the predicted time series for the short-term forecast, the average total per cent

deviation is absolutely less than 20 % in the forecasting period (MGPA), in 25% the absolute MGPA is between 20% and 40%. For about 21 % of the time series the absolute MGPA is greater than 40%.

For which of the time series high total deviations accumulate during the forecasting period? Is it possible to see already in the training process that there will be a strong overestimation or underestimation in the forecasting period? For this purpose, we have analysed the time series that show high deviations and found out that they appear predominantly when there exist many observations in the training period with no demand. In a separate analysis, we therefore have excluded all time series that had at least eight observations less than ten after showing initially high values in the training period. It relates to products that

• in the far-reaching past have belonged to products of demand and function as spare parts in the recent past or

• are being asked for sporadically in great quantities and for which in the following period of time there is zero demand.

For these products, the quantitative forecasting procedures do not really work. Therefore, different procedures should be applied. For the remaining 74 time series, the distribution of the average total percentage deviation is given in figure 3. For about 61% of the forecasted series, the absolute value of the MGPA is less than 20%, for 27% of the time series it is between 20 and 40 % and only for 12% the MGPA is greater than 40%.

Figure 3: MGPA during forecasting period for products with less than eight values smaller than 10 during the training period (3 months)

Deviation	Amount	Per cent	Deviation up to 20 per cent maximum	Deviation up to 40 per cent maximum
[-0.6;-0.4)	1	1.4%		
[-0.4;-0.2)	18	24.3%		
[-0.2;0)	30	40.5%	60.8%	87.8%
[0;0.2)	15	20.3%		
[0.2;0.4)	2	2.7%		
[0.4;∞)	8	10.8%		
Total	74	100.0%		

Even if the forecasting quality ameliorates by restricting the series of demand, deviations of more than 20% are hardly acceptable in practice. In addition, an analysis was made for all time series whether the skewness and kurtosis identified in the training period of the time series are indicators for the forecasting deviation in the forecasting period.

Figure 4 shows the correlation between the classified false forecast – calculated as average total per cent deviation in the forecasting period – and the skewness in the training period between the classified false prognosis and the kurtosis.

Figure 4: Cross-classified tabulation of skewness and kurtosis in the training period and MGPA in the forecasting phase (3 months) – less than eight values smaller than ten

Skewness[1]	MGPA < -0.2	MGPA ϵ [-0.2;0.2]	MGPA ≥ 0.2	Total
Skew	10 (5.6)	11 (13.4)	1 (3.0)	22
Not skew	9 (13.4)	34 (31.6)	9 (7.0)	52
Total	19	45	10	74
Kurtosis[1]	MGPA < -0.2	MGPA ϵ [-0.2;0.2]	MGPA ≥ 0.2	Total
Tend to have no outliers	5 (9.8)	26 (23.1)	7 (5.1)	38
Tend to have outliers	14 (9.2)	19 (21.9)	3 (4.9)	36
Total	19	45	10	71

[1] In parenthesis the respectively expected frequency in case of independence

Regarding the skewness (p=0.03) and the kurtosis (p=0.03) of the forecasting errors in the training period, there is a significant correlation to the MGPA in the forecasting period.

Skewed – and here especially right-skewed distributions – tend to have strong underestimations in

the forecasting period. This can be explained by strong upward outliers that are due to the smoothing characteristics of the models which appear in a short forecasting period of only three months. Distributions that are affected by outliers tend to be underestimated significantly (cf. Kurtosis in figure 4). Non-skewed distributions however tend to have good forecasts with an MGPA below 20%, but are also prone to overestimations. However, 50% of the skewed distributions have an MGPA of over 20%, whereas of the non-skewed distributions only about 35% have an MGPA of over 20%. Biases are very likely for short-term forecasts if there are skewed distributions of the error terms. For medium-term and long-term forecasts, such effects are not being observed, because the potential outliers level is out over such a long forecasting period.

Medium-term Forecasts

Figure 5 shows the distribution for the model with the lowest Theil`s inequality coefficient for the medium-term forecast during the training phase, for which the Portmanteau test gives a positive result. For two time series, the naïve reference model turns out to be the best adaptation. These time series will be excluded from analysis. In the following, we refer to a population of 108 time series.

Model 4 (40. 7%) is used in most cases for classification followed by model 2 (29.6%). Furthermore, figure 5 shows the quality of the forecast in the test phase. Only in 61.1% of the models, Theil`s inequality coefficient is less than one, that is the forecast is better than the naïve adjustment of assessed values. 56.1% of the ARIMA (p,d,q) models without level (model 4), and 64.3% of the ARIMA (p,d,q) models with level (model 7) lie below 1 with their Theil`s inequality coefficient of the forecast phase. For the pure MA (q) models (model 2) it is only 62.5% for the medium-term forecast.

Figure 5: **Models with the best forecast quality from the training data applied to the test data set (6-month forecast)**

Type of forecast	Forecast model	Chosen[1]	Worse than reference model		Better than reference model	
			Amount	Per cent	Amount	Per cent
Time series econometric procedure	Model 2	29.6%	12	37.5%	20	62.5%
	Model 3	3.7%	2	50.0%	2	50.0%
	Model 4	40.7%	18	43.9%	23	56.1%
	Model 5	3.7%	2	50.0%	2	50.0%
	Model 6	5.6%	1	16.7%	5	83.3%
	Model 7	13.0%	5	35.7%	9	64.3%
Structural regression	Model 8	2.8%	2	66.7%	1	33.3%
	Model 9	1.9%	0	.0%	0	.0%
Mixed procedures	Model 10	4.6%	0	.0%	4	100.0%
	Total		42	38.9%	66	61.1%

[1] Due to multiple answers there is a total sum of more than 110 time series and a percentage sum of more than 100%.

We again exclude time series for which - during the training period - eight or more values are smaller than ten. The distribution of the total medium deviation in per cent is shown in figure 6. For 66% of the forecasted series, the MGPA is less than 20% in absolute terms, for 18% of the time series the MGPA is between 20% and 40% and for 16% of the time series it is greater than 40 %.

Figure 6: MGPA in the forecasting period for products with less than eight values smaller than ten in the training phase

Deviation	Amount	Per cent	Deviation up to max. 20%	Deviation up to max. 40 per cent
[-0.6;-0.4)	2	2.6%		
[-0.4;-0.2)	3	3.9%		
[-0.2;0)	22	28.9%	65.7%	84.1%
[0;0.2)	28	36.8%		
[0.2;0.4)	11	14.5%		
[0.4;0.6)	10	13.1%		
Total	76	100.0%		

The skewness (p=0.256) and kurtosis (p=0.054) of the forecasting errors during the training period do not show a significant correlation to the average total deviation in per cent. Thus, outliers do not lead to the above-average error forecasts. This could also be a reason why the part of forecasts with an MGPA less than 20% is five percentage points higher than for short-term forecasts. It is questionable to take this as a reference for the adaptation quality, because the part of forecasts with considerable errors of more than 40% for the medium-term forecasts is four percentage points higher than for short-term forecasts.

Long-term Forecasts

For the long-term forecast, the ARIMA (p,d,q) models (38%) and the pure MA(q) model (25.9%) are the forecasting models with the best quality in the training data set. Two time series could not be adjusted, because the residuals were not considered as White Noise. Therefore, we refer again to a population of 108 time series. Figure 7 shows the quality of the forecast for the application to the test data set. Also, for the long-term forecast the ARIMA(p,d,q) model (model 4) with 59.5% of forecasting measures smaller than one, and the ARIMA(p,d,q)- model with level (model 7) with 60% come off well.

Figure 7: Models with best forecasting quality from the training data set applied to the test data set (12 months forecast)

Forecast type	Forecast model	Chosen[1]	Worse than reference model		Better than reference model	
			Amount	Per cent	Amount	Per cent
Time series econometric procedure	Model 2	25.9%	12	42.9%	16	57.1%
	Model 3	5.6%	3	50.0%	3	50.0%
	Model 4	38.0%	15	40.5%	22	59.5%
	Model 5	4.6%	2	40.0%	3	60.0%
	Model 6	9.3%	2	20.0%	8	80.0%
	Model 7	13.9%	6	40.0%	9	60.0%
Structural regression	Model 8	4.6%	3	60.0%	2	40.0%
	Model 9	1.9%	0	0.0%	1	100.0%
Mixed procedure	Model10	0.9%	0	0.0%	1	100.0%
	Total		43	39.81%	65	60.19%

[1] Due to multiple answers there is a total sum of more than 110 time series and a percentage sum of more than 100%.

Figure 8 shows the distribution of the average deviation in percent of the 78 time series for which less than eight values are less than ten during the training period. Here 60% of the deviations have an absolute value less than 20%, for 31% the absolute value of deviations is between 20% and 40%, only 9% of deviations are absolutely greater than 40%.

Figure 8: MGPA during forecasting period for products with less than eight values smaller than ten in the training period (12 months)

Deviation	Amount	Per cent	Deviation up to max. 20 per cent	Deviation up to max 40 per cent
[-1;-0.4)	2	2.6%		
[-0.4;-0.2)	4	5.1%		
[-0.2;0)	26	33.3%	60.2%	90.9%
[0;0.2)	21	26.9%		
[0.2;0.4)	20	25.6%		
[0.4;0.8)	5	6.4%		
Total	78	100.0%		

Therefore, the forecasting quality corresponds with the short-term forecasts: However, long-term forecasts do not have an MGPA of less than 20% as often as medium-term forecasts, but they seldom show an MGPA forecasting error of more than 40%. Also, the above-average underestimation of skewed distributions and the above-average overestimation of non-skewed

distributions are analogous to the structure of short-term forecasts. The p-value of the skewness is p=0.04, whereas the amount of cells with an expected frequency less than five is marginally high with 33% and biases the significance of the results. For the kurtosis, there is no significant dependence of the MGPA from the outlier proneness of the time series (p=0.160).

Figure 9: Cross-tabulation of skewness in the training phase and MGPA in the forecasting phase (12 months)(less than eight values smaller than ten)

Skewed[1]	MGPA < -0.2	MGPA ϵ [-0.2;0.2]	MGPA $\geq$ 0.2	Total
Skewed	5 (2.5)	20 (19.3)	7 (10.3)	32
Not skewed	1 (3.5)	27 (27.7)	18 (14.7)	46
Total	6	47	25	78

[1] In parenthesis the expected frequency for independence

CONCLUSION

Randomly fluctuant demand for feedstock makes the use of stochastic models in inventory management necessary. In the presented algorithmic approach based on the data of a German engineering company, an optimal forecasting model will be automatically calculated by means of criteria closeness, which will be used for the forecasting of demand. For all the chosen forecasting periods, the ARMA-model of the d-differentiated time series turned out to be the best forecasting model, followed by simple Moving Average and ARIMA models (cf. figure 10). The significance of autoregressive procedures diminishes with the length of the forecasting period. Structural approaches almost never turn out to be the best forecasting models. Even if their quality increases with the length of the forecasting period, only about 7% of the time series for structural models show the best forecast for the 12-month period. We therefore propose the forecasting of demand by the ARMA-model of the d-differentiated time series which –

as a sensitivity analysis - could be complemented by forecasts with the simple Moving Average model and eventually the ARIMA model with level.

The forecasting of results without further sampling criteria were not satisfying in the beginning, for 20% of the products there was an average total per cent deviation of more than 40% for the short-term forecasting period. Responsible for this are the time series with discontinued and sporadic demand. The "smoothing" characteristics of the forecasting procedures are hardly able to forecast sporadic monthly demands with value zero. For the important and current demand products, such "zero-demands" are practically non-existent, so that the problem arises only for the products, which are only demanded as spare parts due to antiquated technology. The more a continuously high demand of a product in the past is replaced by an unsteady and low demand in the near future, the forecasting results turn out to be of low quality. Restricting the time series to those that have at least eight observations with a demand of less than ten products, the

MGPA-rate greater than 40% for short-term forecasts dropped from 20% to 12%, for medium-term forecasts to 16% and for long-term forecasts down to 9%.

Forecast period	Absolute value of MGPA			MGPA< (-20%)	Share of time series with forecast better than reference model	Best three models	Per cent of struct. models
	[0;20%[	[20%;40%[	[40%;∞]				
Short-term	61%	27%	12%	26%	62%	4 (46%) 2 (22%) 7 (19%)	1.8%
Medium-term	66%	18%	16%	7%	61%	4 (41%) 2 (30%) 7 (13%)	4.7%
Long-term	60%	31%	9%	8%	61%	4 (38%) 2 (26 %) 7 (14%)	6.5%

Figure 10: Summary of the results

A comparison of the different forecasting periods shows that the amount of well forecasted values (absolute value of MGPA is less than 20%) with 66% for medium-term forecasts (short-term 61%, long-term 60%) are the highest. However, the rather bad forecasted values (absolute value of MGPA is greater than 40%) for the medium-term forecast are with 16% the highest (short-term 12%, long-term 9%). If one takes for the practical materials planning an MGPA less than 20% as tolerable limiting value, then one is able to forecast in a satisfying way at least 60% of all product groups with the help of the described procedures. The forecasts of the remaining 40% should be interpreted with utmost caution. A more profound analysis of the reasons for such great deviations is necessary. A first analysis shows that it can be related to very heterogeneous product groups that hide behind their product number certain aggregates of products. Behind the term "condenser or capacitor" there are not specific ones, but a number of different types and technology generations of condensers/capacitors. Individual condenser types can fluctuate out of completely different reasons in this product group very much and cause an unexpected - and thus unpredictable – course of the time series. The limitation of forecasting to homogeneous product groups increases the reliability of forecasting in a significant way.

It can be seen that with increasing forecasting period, the values will be more over- than underestimated. For the short-term forecasts, there are 26% of the deviations 20% or more too low, for the medium-term forecasts it is only 7% and for the long-term forecasts 8%, whereas deviations over 20% upward for short-term forecasts account for 11%, for medium-term forecasts 13% and for long-term forecasts 13%.

All in all, the described procedures of an algorithmic application of the different quantitative forecasting

techniques should be applied only if the following facts are given:

- Forecasts should only be applied for disaggregated – that is for homogeneous groups of products. If product aggregates are used, it should be assured that individual product types do not develop too heterogeneously in the time series.

- The MGPA of the time series should not exceed the value of 20% in the past. It should be controlled continuously.

- Time series with sporadic zero-demand or sporadic outliers upwards (e.g. procuring replacement of expiring products) sometimes give a good forecast by chance. Nevertheless, the demand of this time series should be forecasted with other procedures.

- Skew distributions should be applied with care. The risk of clearly underestimating the demand is great – especially for short-term forecasts.

Considering these prerequisites then the algorithmic procedure - easy to calculate by computer - can simplify the practical task of forecasting the outflow of goods for a considerable part of consumable supplies. Even though the results seem satisfying from the methodical point of view, the decision-makers in a company have to examine if one can rely only on the presented algorithm concerning material requirements planning. Based on the fact that lacking material can stop entire production processes, the question has to be answered with a definite `no`. The algorithm can only be one out of many other means in the supply chain management. It has to be combined with additional information from within the company (e.g. from the Sales Department etc.) and outside the company (e.g. about delivery time etc.). However, these aspects have not been considered in this article, but they are important aspects for decisions in supply chain management.

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